NRL Report 6321

Directional Equilibrium of an Artificial Satellite Enveloping a Nonuniform Region of a Gravitational Field

P. A. CRAFTON

Security Systems and Avigation Branch
Electronics Division

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U.S. NAVAL RESEARCH LABORATORY
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ABSTRACT

Certain artificial satellites used for communications and/or navigation require means of directional stabilization. If the satellite encompasses an essentially uniform region of the gravitational field, it is in neutral directional equilibrium in that force field. The creation of an artificial satellite with a discrete distribution of mass permits us to have a satellite encompassing a nonuniform region of the gravitational field although having a relatively low total mass.

The basic mathematical relationships are developed that must be satisfied by any "gravity-gradient" satellite in order that it have stable directional equilibrium. The satellite is assumed to have a discrete distribution of mass, and is assumed to be not subjected to other body forces, radiation forces, and disturbing forces.

PROBLEM STATUS

This is an interim report on one phase of the problem. Research is continuing on this and other phases of the problem.

AUTHORIZATION

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DIRECTIONAL EQUILIBRIUM OF AN ARTIFICIAL SATELLITE ENVELOPING A NONUNIFORM REGION OF A GRAVITATIONAL FIELD

INTRODUCTION

The "gravity-gradient" artificial satellite depends for stable directional equilibrium on its encompassing a discernibly nonuniform region of the gravitational field. Most feasibly, this is done by a discrete distribution of mass sufficiently removed from the satellite proper. Particles are rigidly coupled to the satellite proper by relatively massless but long rigid rods. If the overall span of the total satellite is of the order of say 100 feet, then there exists a small but useful nonuniformity in the gravitational field intensity over the satellite span.

This report is written in terms of an undefined number of rod-particle pairs. The analysis assumes that the dimensional extent of the satellite proper (total satellite sans rods and particles) is small compared to the dimensional extent of the total satellite, and that the gravitational field is constant over the extent of the satellite proper; the satellite proper may therefore be considered a particle. The center of mass of the total satellite, however, is not assumed to be at the center of mass of the satellite proper, although the center of mass may lie within the boundaries of the latter depending on the contribution of the added particles to the total distribution of mass.

The total satellite is in stable orbital equilibrium in a gravitational field whose magnitude obeys the inverse-square law. Other body forces, radiation forces, and all disturbing forces are considered to be negligible.

DYNAMICS OF THE SATELLITE

The satellite is essentially a compound pendulum without a constrained point of rotation. Its center of mass translates so as to describe a stable orbit, and any rotational motion is about its center of mass. In the absence of an initial undamped rotation, such rotational motion can be caused only by a moment of force about the center of mass. The moment of force is due only to the presence of the particles of the satellite in the gravitational field, and is

$$\vec{N} = \sum_{\alpha} \vec{r}\,'_{(\alpha)} \times m_{(\alpha)}\, \vec{g}_{(\alpha)} \tag{1}$$

where $\vec{r}\,'_{(\alpha)}$ is the position vector of the αth particle (including the satellite proper) originating at the center of mass, $m_{(\alpha)}$ is the mass of the αth particle, and $\vec{g}_{(\alpha)}$ is the intensity of the gravitational field at the point occupied by the αth particle. If the force field were uniform and its intensity $\vec{g}$, then we would have

$$\vec{N} = \sum_{\alpha} \vec{r}\,'_{(\alpha)} \times m_{(\alpha)}\, \vec{g} = \left(\sum_{\alpha} m_{(\alpha)}\, \vec{r}\,'_{(\alpha)}\right) \times \vec{g} \equiv 0 \tag{2}$$

since

$$\sum_{\alpha} m_{(\alpha)}\, \vec{r}\,'_{(\alpha)} \equiv 0 \tag{3}$$

by the definition of the center of mass. Thus a uniform field cannot cause rotation about the center of mass.

If we call $\vec{R}$ the position vector of the center of mass and $\vec{r}_{(\alpha)}$ the position vector of the αth particle, each position vector originating at the geocenter, we have

$$\vec{r}\,'_{(\alpha)} \equiv \vec{r}_{(\alpha)} - \vec{R} \tag{4}$$

and therefore

$$\vec{N} = \sum_{\alpha} (\vec{r}_{(\alpha)} - \vec{R}) \times m_{(\alpha)} \vec{g}_{(\alpha)} = \sum_{\alpha} \vec{r}_{(\alpha)} \times m_{(\alpha)} \vec{g}_{(\alpha)} - \vec{R} \times \sum_{\alpha} m_{(\alpha)} \vec{g}_{(\alpha)} \tag{5}$$

where

$$\sum_{\alpha} m_{(\alpha)} \vec{g}_{(\alpha)}$$

is the local weight of the total satellite.

If the gravitational field is point symmetrical about the geocenter, then $\vec{r}_{(\alpha)} \times \vec{g}_{(\alpha)} \equiv 0$. Hence

$$\sum_{\alpha} \vec{r}_{(\alpha)} \times m_{(\alpha)} \vec{g}_{(\alpha)} \equiv 0 \tag{6}$$

and we have

$$\vec{N} = -\vec{R} \times \sum_{\alpha} m_{(\alpha)} \vec{g}_{(\alpha)} . \tag{7}$$

Under this condition

$$\vec{g}_{(\alpha)} = \frac{-\gamma \vec{r}_{(\alpha)}}{|\vec{r}_{(\alpha)}|^3} = \frac{-\gamma (\vec{r}\,'_{(\alpha)} + \vec{R})}{|\vec{r}\,'_{(\alpha)} + \vec{R}|^3} \tag{8}$$

where γ is a constant of the field. Hence

$$\begin{aligned} \vec{N} &= \vec{R} \times \gamma \sum_{\alpha} \frac{m_{(\alpha)} (\vec{r}\,'_{(\alpha)} + \vec{R})}{|\vec{r}\,'_{(\alpha)} + \vec{R}|^3} \\ &= \vec{R} \times \gamma \sum_{\alpha} \frac{m_{(\alpha)} \vec{r}\,'_{(\alpha)}}{|\vec{r}\,'_{(\alpha)} + \vec{R}|^3} + \vec{R} \times \gamma \sum_{\alpha} \frac{m_{(\alpha)} \vec{R}}{|\vec{r}\,'_{(\alpha)} + \vec{R}|^3} \end{aligned} \tag{9}$$

where the last term of Eq. (9) identically vanishes. Thus

$$\vec{N} = \vec{R} \times \gamma \sum_{\alpha} \frac{m_{(\alpha)} \vec{r}\,'_{(\alpha)}}{|\vec{r}\,'_{(\alpha)} + \vec{R}|^3} . \tag{10}$$

We cannot make the approximation $|\vec{r}\,'_{(\alpha)} + \vec{R}| \approx |\vec{R}|$, for to do so would make

$$\vec{N} = \vec{R} \times \gamma \sum_{\alpha} \frac{m_{(\alpha)} \vec{r}\,'_{(\alpha)}}{|\vec{R}|^3} \equiv 0 .$$

COORDINATE SYSTEMS

Figure 1 shows the geocentric celestial sphere centered at the origin of the Euclidean reference frame x. The x^3-plane through the origin is the equatorial plane of the earth. The x^1-axis is directed at some reference, say the first star of Aries. The spherical coordinates of the satellite are the u^i, where $u^1 = |\vec{R}|$, u^2 is the right ascension, and u^3 is the declination of the satellite.

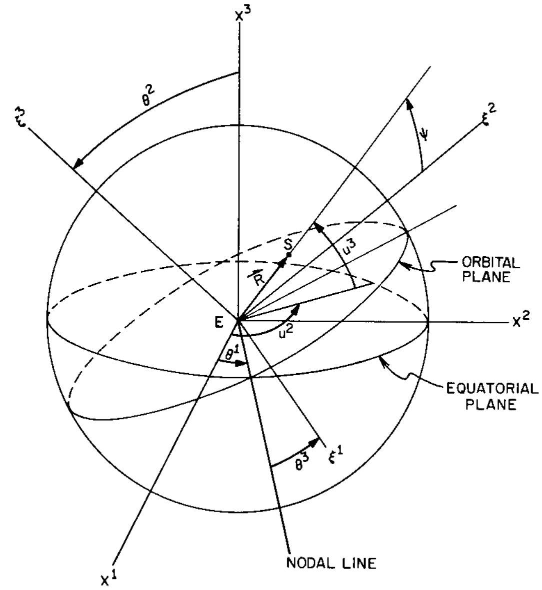

Fig. 1 - Geocentric celestial sphere

In order to determine the directional positions of equilibrium of the satellite, we need a meaningful reference coordinate system. For this purpose we will use the ζ Euclidean system (shown in Fig. 2) originating at the center of mass of the satellite. Axis ζ^1 is colinear with $\vec{R}$, axis ζ^2 is tangent to the curve of constant declination through the center of mass, and axis ζ^3 is tangent to the curve of constant right ascension through the center of mass. The coordinate system ζ therefore translates with the satellite but does not rotate about its center of mass. The geocentric coordinate system z remains parallel to ζ and therefore rotates about the geocenter as the satellite revolves about its primary.

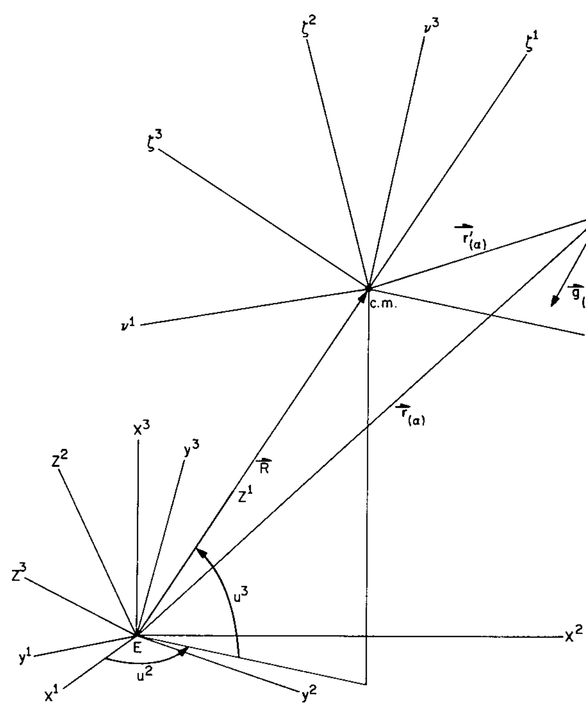

Fig. 2 - Satellite coordinate systems

We additionally need a centroidal body Euclidean coordinate system ν in which the distribution of mass is known and constant. The orientation of ν relative to the body is arbitrary but fixed. The corresponding parallel geocentric coordinate system is y which rotates about the geocenter as the satellite rotates about its center of mass. The rotational motion of the satellite about is center of mass and the rotational positions of equilibrium are stated by the Eulerian angles φ^i between y and z, which are the same angles between ν and ζ.

The equations of coordinate transformation between z and the reference frame x are

$$z^i = A^i_j\, x^j \tag{11}$$

where

$$(A^i_j) = \begin{pmatrix} \cos u^2 \cos u^3 & \sin u^2 \cos u^3 & \sin u^3 \\ -\sin u^2 & \cos u^2 & 0 \\ -\cos u^2 \sin u^3 & -\sin u^2 \sin u^3 & \cos u^3 \end{pmatrix} \tag{12}$$

in terms of the right ascension and the declination of the satellite.

DISTRIBUTION OF MASS

The discrete distribution of mass may be expressed in terms of the spherical coordinates of each particle in a spherical coordinate system w originating at the center of mass and defined with reference to the body coordinate system ν (see Fig. 3). When we express the position vectors as

$$\vec{r}\,'_{(\alpha)} = \vec{c}_i\, \nu^i_{(\alpha)} \tag{13}$$

where the $\vec{c}_i$ are the base vectors of the y or ν coordinate system, we have

$$\left.\begin{aligned} \nu^1_{(\alpha)} &= w^1_{(\alpha)} \cos w^2_{(\alpha)} \cos w^3_{(\alpha)} \\ \nu^2_{(\alpha)} &= w^1_{(\alpha)} \sin w^2_{(\alpha)} \cos w^3_{(\alpha)} \\ \nu^3_{(\alpha)} &= w^1_{(\alpha)} \sin w^3_{(\alpha)} \end{aligned}\right\} . \tag{14}$$

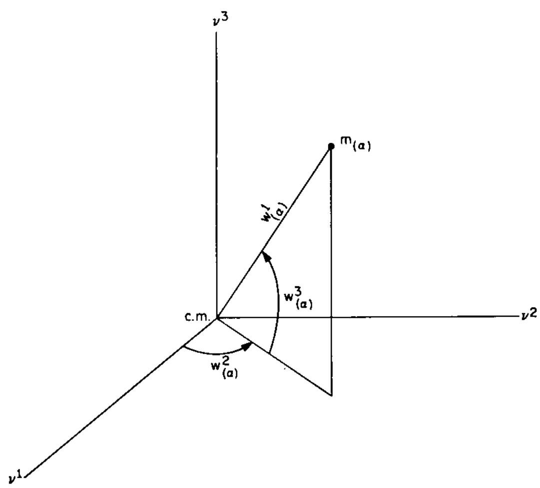

Fig. 3 - Distribution of mass

The moment of force is therefore

$$\vec{N} = \gamma \vec{R} \times \sum_{\alpha} \frac{m_{(\alpha)}\, \vec{c}_i\, \nu^i_{(\alpha)}}{\left|\vec{c}_i\, \nu^i_{(\alpha)} + \vec{R}\right|^3} \tag{15}$$

where the $\nu^i_{(\alpha)}$ are constants, but the $\vec{c}_i$ are functions of the φ^i. The $\vec{c}_i$ are of unit magnitude but they rotate with the satellite about its center of mass and therefore are variable from the point of view of the reference frame x or of the rotational-reference coordinate system z. We have

$$\vec{c}_i(y) = \vec{a}_j(z) \frac{\partial z^j}{\partial y^i} \tag{16}$$

where the $\vec{a}_j$ are the base vectors of the z or ζ coordinate system, and the $\partial z^j / \partial y^i$ are functions of the Eulerian angles φ^i. Also,

$$\vec{a}_j(z) = \vec{e}_k(x) \frac{\partial x^k}{\partial z^j} \tag{17}$$

where the $\vec{e}_k$ are the base vectors of the x coordinate system and the $\partial x^k / \partial z^j$ depend on the satellite's right ascension and declination. Hence,

$$\vec{c}_i(y) = \vec{e}_k \frac{\partial x^k}{\partial z^j} \frac{\partial z^j}{\partial y^i} \tag{18}$$

SATELLITE ORBIT

If we assume a point-symmetrical gravitational field about the geocenter and if we also assume the absence of other body forces, of radiation forces, and of disturbing forces, then the satellite orbit is an ellipse with one focus at the geocenter:

$$|\vec{R}| = \frac{a[1 - (\epsilon)^2]}{1 + \epsilon \cos \psi} \tag{19}$$

where a is the major semiaxis of the ellipse, ϵ is its eccentricity, and $\psi(t)$ is the angle between $\vec{R}$ and the axis ξ^2 shown in Fig. 4. ξ is the geocentric Euclidean coordinate system of the orbit. The ξ^3-plane is the orbital plane; the ξ^2-axis is the major axis; and the ξ^1-axis is the minor axis. Let the η^i be the base vectors of ξ, so that

$$\vec{R} = \vec{\eta}_i \, \xi^i_{(c)} \tag{20}$$

where the $\xi^i_{(c)}$ are the coordinates of the center of mass of the satellite. Hence,

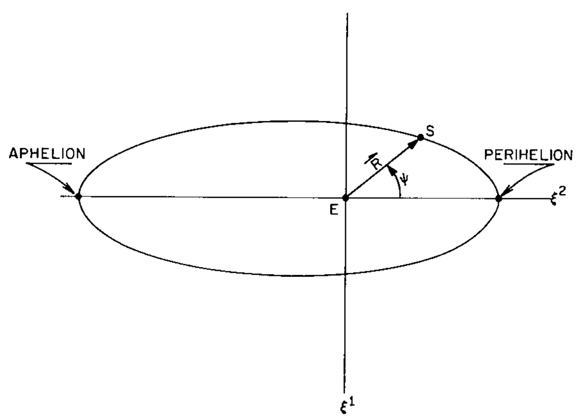

Fig. 4 - Satellite orbit

$$\left.\begin{aligned} \xi^1_{(c)} &= |\vec{R}| \cos\left(\psi + \frac{\pi}{2}\right) = \frac{a[1-(\epsilon)^2]\cos\left(\psi+\frac{\pi}{2}\right)}{1+\epsilon\cos\psi} \\ \xi^2_{(c)} &= |\vec{R}| \sin\left(\psi + \frac{\pi}{2}\right) = \frac{a[1-(\epsilon)^2]\sin\left(\psi+\frac{\pi}{2}\right)}{1+\epsilon\cos\psi} \\ \xi^3_{(c)} &= 0 \end{aligned}\right\} . \tag{21}$$

Figure 1 also shows the three rotational orbital elements θ^i of the satellite: θ^1 is the angle between x^1 and the nodal line; θ^2 is the angle between the orbital plane and the equatorial plane; and θ^3 is the perihelion constant. (The remaining three, and nonrotational, orbital elements are a, ϵ, and the instant of time at which the satellite is at its perihelion.)

The Euclidean coordinates $x^i_{(c)}$ of the satellite's center of mass are functions of the ξ^i and the θ^i:

$$x^i_{(c)} = x^i_{(c)}(\xi^1_{(c)}, \xi^2_{(c)}, \theta^1, \theta^2, \theta^3) . \tag{22}$$

The θ^i are constants, but the ξ^i are functions of time inasmuch as ψ is a function of time. We have

$$x^i_{(c)} = D^i_j \, \xi^j_{(c)} \qquad (i = 1, 2, 3; \; j = 1, 2) \tag{23}$$

where

$$\left.\begin{aligned} D^1_1 &= \cos\theta^1 \cos\theta^3 - \sin\theta^1 \cos\theta^2 \sin\theta^3 \\ D^1_2 &= -\cos\theta^1 \sin\theta^3 - \sin\theta^1 \cos\theta^2 \cos\theta^3 \\ D^2_1 &= \sin\theta^1 \cos\theta^3 + \cos\theta^1 \cos\theta^2 \sin\theta^3 \\ D^2_2 &= -\sin\theta^1 \sin\theta^3 + \cos\theta^1 \cos\theta^2 \cos\theta^3 \\ D^3_1 &= \sin\theta^2 \sin\theta^3 \\ D^3_2 &= \sin\theta^2 \cos\theta^3 \end{aligned}\right\} . \tag{24}$$

The corresponding inverse equations of coordinate transformation are

$$\xi^i_{(c)} = E^i_j \, x^i_{(c)} \qquad (i = 1, 2; \; j = 1, 2, 3) \tag{25}$$

where

$$\left.\begin{aligned} E^1_1 &= \cos\theta^1 \cos\theta^3 - \sin\theta^1 \cos\theta^2 \sin\theta^3 \\ E^1_2 &= \sin\theta^1 \cos\theta^3 + \cos\theta^1 \cos\theta^2 \sin\theta^3 \\ E^1_3 &= \sin\theta^2 \sin\theta^3 \\ E^2_1 &= -\cos\theta^1 \sin\theta^3 - \sin\theta^1 \cos\theta^2 \cos\theta^3 \\ E^2_2 &= -\sin\theta^1 \sin\theta^3 + \cos\theta^1 \cos\theta^2 \cos\theta^3 \\ E^2_3 &= \sin\theta^2 \cos\theta^3 \end{aligned}\right\} \tag{26}$$

in which the θ^i are the rotational orbital elements of the satellite and are constants independent of time in the absence of disturbing forces.

ROTATIONAL EQUILIBRIUM OF THE SATELLITE

The satellite is in rotational equilibrium whenever $\vec{N}(\varphi^1, \varphi^2, \varphi^3)$ vanishes. The equilibrium is stable if $\partial\vec{N}/\partial\varphi^i < 0$ for the set of zeros of $\vec{N}(\varphi^1, \varphi^2, \varphi^3)$ defining the position of equilibrium, and is unstable if $\partial\vec{N}/\partial\varphi^i > 0$.

The basic equation for the case of the point-symmetrical field is Eq. (15) and is

$$\begin{aligned}\vec{N} &= \gamma\vec{R} \times \sum_\alpha \frac{m_{(\alpha)}\,\vec{c}_i\,\nu^i_{(\alpha)}}{\left|\vec{c}_i\,\nu_{(\alpha)} + \vec{R}\right|^3} \\ &= \gamma\vec{R} \times \sum_\alpha \frac{m_{(\alpha)}\,\nu^i_{(\alpha)}\,\vec{e}_k\,\dfrac{\partial x^k}{\partial z^j}\,\dfrac{\partial z^i}{\partial y^i}}{\left|\vec{e}_k\,\dfrac{\partial x^k}{\partial z^j}\,\dfrac{\partial z^j}{\partial y^i}\,\nu^i_{(\alpha)} + \vec{e}_k x^k_{(c)}\right|^3}\end{aligned} \tag{27}$$

where

$$\left.\begin{aligned} x^1_{(c)} &= u^1 \cos u^2 \cos u^3 \\ x^2_{(c)} &= u^1 \sin u^2 \cos u^3 \\ x^3_{(c)} &= u^1 \sin u^3 \end{aligned}\right\}. \tag{28}$$

The components of the moment of force in the x coordinate system are therefore

$$\begin{aligned} N^i &= \gamma\varepsilon^i_{j\,k}x^j_{(c)} \sum_\alpha \frac{m_{(\alpha)}\,\nu^n_{(\alpha)}\,\dfrac{\partial x^k}{\partial z^s}\,\dfrac{\partial z^s}{\partial y^n}}{\left|\vec{e}_k\left[\dfrac{\partial x^k}{\partial z^j}\,\dfrac{\partial z^j}{\partial y^i}\,\nu^i_{(\alpha)} + x^k_{(c)}\right]\right|^3} \\ &= \gamma\varepsilon^i_{j\,k}x^j_{(c)} \sum_\alpha \frac{m_{(\alpha)}\nu^n_{(\alpha)}\,\dfrac{\partial x^k}{\partial z^s}\,\dfrac{\partial z^s}{\partial y^n}}{\left\{\delta_{pq}\left[\dfrac{\partial x^p}{\partial z^s}\,\dfrac{\partial z^s}{\partial y^n}\,\nu^n_{(\alpha)} + x^p_{(c)}\right]\left[\dfrac{\partial x^q}{\partial z^s}\,\dfrac{\partial z^s}{\partial y^n}\,\nu^n_{(\alpha)} + x^q_{(c)}\right]\right\}^{3/2}} \end{aligned} \tag{29}$$

where δ_{pq} is the Kronecker delta, $\varepsilon^i_{j\,k}$ is the generalized Kronecker delta, and the $\partial z^j/\partial y^i$ are defined by*

*See Eq. (40) in P. A. Crafton, "On the Determination of the Deck Motion of Aircraft Carriers," NRL Report 6215, July 1965.

$$\left(\frac{\partial z^j}{\partial y^i}\right) = \begin{pmatrix} \cos\varphi^1\cos\varphi^3 - \sin\varphi^1\cos\varphi^2\sin\varphi^3 & -\cos\varphi^1\sin\varphi^3 - \sin\varphi^1\cos\varphi^2\cos\varphi^3 & \sin\varphi^1\sin\varphi^2 \\ \sin\varphi^1\cos\varphi^3 + \cos\varphi^1\cos\varphi^2\sin\varphi^3 & -\sin\varphi^1\sin\varphi^3 + \cos\varphi^1\cos\varphi^2\cos\varphi^3 & -\cos\varphi^1\sin\varphi^2 \\ \sin\varphi^2\sin\varphi^3 & \sin\varphi^2\cos\varphi^3 & \cos\varphi^3 \end{pmatrix} . \tag{30}$$

The $\partial x^k/\partial z^j$ in Eq. (29) are defined by Eqs. (11) as functions of the right ascension and the declination. The $x^i_{(c)}$ are the coordinates of the satellite and are defined by Eqs. (23) in terms of $\psi(t)$, the eccentricity of the orbit, and the major semiaxis.

Thus the N^i are explicitly functions of time, since the $x^i_{(c)}$ are functions of time. But they are also explicitly functions of the Eulerian angles φ^i, since the $\partial z^j/\partial y^i$ are functions of the φ^i. The positions of rotational equilibrium will therefore vary with time for an elliptic orbit. For each point of the orbit, the N^i may be considered to be functions of the φ^i alone. The positions of rotational equilibrium are therefore found by a solution of the system of equations

$$N^i(\varphi^1, \varphi^2, \varphi^3) = 0 . \tag{31}$$

Each position of equilibrium is stable if the

$$\frac{\partial N^i}{\partial \varphi^j} < 0 \qquad (i, j = 1, 2, 3) \tag{32}$$

for the set of zeros of $N^i(\varphi^1, \varphi^2, \varphi^3)$ and is unstable if the

$$\frac{\partial N^i}{\partial \varphi^j} > 0 \qquad (i, j = 1, 2, 3)$$

for the set of zeros of $N^i(\varphi^1, \varphi^2, \varphi^3)$.

We find the $\partial N^i/\partial \varphi^j$ from Eq. (29) to obtain

$$\begin{aligned} \frac{\partial N^i}{\partial \varphi^j} = \gamma \epsilon^i_{\beta k}\, x^\beta \sum_a \left[m_{(a)}\, \nu^n_{(a)} \frac{\partial x^k}{\partial z^s} \right] & \cdot \left(\frac{1}{\left\{ \delta_{pq} \left[\frac{\partial x^p}{\partial z^r} \frac{\partial z^r}{\partial y^b} \nu^b_{(a)} + x^p_{(c)} \right] \left[\frac{\partial x^q}{\partial z^m} \frac{\partial z^m}{\partial y^l} \nu^l_{(a)} + x^q_{(c)} \right] \right\}^{5/2}} \right) \\ & \cdot \left\{ \frac{\partial^2 z^s}{\partial y^n \partial \varphi^j} \delta_{pq} \left[\frac{\partial x^p}{\partial x^r} \frac{\partial z^r}{\partial y^b} \nu^b_{(a)} + x^p_{(c)} \right] \left[\frac{\partial x^q}{\partial z^m} \frac{\partial z^m}{\partial y^l} \nu^l_{(a)} + x^q_{(c)} \right] \right. \\ & \left. - 3\,\delta_{pq} \left[\frac{\partial x^p}{\partial z^r} \frac{\partial x^q}{\partial z^m} \frac{\partial^2 z^m}{\partial y^l \partial \varphi^j} \frac{\partial z^r}{\partial y^b} \nu^l_{(a)} \nu^b_{(a)} + \frac{\partial x^q}{\partial z^m} \frac{\partial^2 z^m}{\partial y^l \partial \varphi^j} x^p_{(c)} x^l_{(c)} \right] \right\} . \end{aligned} \tag{33}$$

CONCLUDING REMARKS

An analysis has been presented giving the mathematical relationships that must be satisfied by a "gravity-gradient" satellite in order that it have a position or positions of rotational stable equilibrium about its center of mass. The final equations are for a satellite with a discrete distribution of mass moving in an orbitally stable path in a gravitational field symmetrical about the geocenter. Numerical results would depend on the actual mass distribution of the satellite and on its actual orbital elements.

* * *

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13. ABSTRACT

Certain artificial satellites used for communications and/or navigation require means of directional stabilization. If the satellite encompasses an essentially uniform region of the gravitational field, it is in neutral directional equilibrium in that force field. The creation of an artificial satellite with a discrete distribution of mass permits us to have a satellite encompassing a nonuniform region of the gravitational field although having a relatively low total mass.

The basic mathematical relationships are developed that must be satisfied by any "gravity-gradient" satellite in order that it have stable directional equilibrium. The satellite is assumed to have a discrete distribution of mass, and is assumed to be not subjected to other body forces, radiation forces, and disturbing forces.

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